Assignment 4

Coverage: 15.5 in Text.

Exercises: 15.5. no 3, 4, 21, 24, 25, 27, 29, 32, 33, 38, 39, 42.

Submit 15.5 no. 24, 27, 29, and 33 by Feb 14.

Supplementary Problems

1. Find the equations of the planes passing through the origin and (a) $(1,2,3),(0,-2,0)$ and (b) $(0,2,-1),(3,0,5)$.
2. Find the equation of the plane passing the points $(1,0,-1),(4,0,0),(6,2,1)$.

The Equation of a Plane

The equation of a plane in space is in the form

$$ax+by+cz=d\ ,$$

and $d=0$ if and only if the plane passes through the origin. Given three points in space $\mathbf{0}=(0,0,0),\mathbf{u}_1=(x_1,y_1,z_1),\mathbf{u}_2=(x_2,y_2,z_2)$, the equation of the plane can be determined by the following formula:

$$(a,b,c)=\mathbf{u}_1\times\mathbf{u}_2\ ,$$

in $ax+by+cz=0$. Here $\times$ is the cross product for vectors.

When the plane does not pass through the origin, the three points are $\mathbf{u}_0,\mathbf{u}_1,\mathbf{u}_2$. Let $\mathbf{v}_1=\mathbf{u}_1-\mathbf{u}_0,\mathbf{v}_2=\mathbf{u}_2-\mathbf{u}_0$. Then

$$(a,b,c)=\mathbf{v}_1\times\mathbf{v}_2\ ,$$

in the equation $ax+by+c=d$. The number d can be obtained by $d=ax_0+by_0+cz_0$ where $\mathbf{u}_0=(x_0,y_0,z_0)$.

Let us look at one example. Find the equation of the plane passing through $(0,0,0),(-1,2,0),(0,0,6)$. The cross product of $(-1,2,0)$ and $(0,0,6)$ is $(12,6,0)$. Hence the equation of this plane is $12x+6y=0$ or $2x+y=0$.

Next, find the equation of the plane passing through $(1,0,1),(0,2,0),(2,0,-3)$. Using $(0,2,0)-(1,0,1)=(-1,2,-1)$ and $(2,0,-3)-(1,0,1)=(1,0,-4)$ and $(-1,2,-1)\times(1,0,-4)=(-8,-5,-2)$. Hence the equation is $-8x-5y-2z=d$. Plugging in $(1,0,1)$, $d=-8-2=-10$, so the equation is $-8x-5y-2z=-10$ or $8x+5y+2z=10$.